

PRESENTATION

MATHIEU DUPONT/BRECKES

The goal of this blog is mainly to advertise two nonstandard points of view on category theory (and, in particular, mathematics) that I've adopted 6 years ago and which gave me a clearer picture of category theory:

- (1) one should define categories without an equality at the level of objects (so that equality is defined only between arrows between two fixed objects);
- (2) the right higher-dimensional hierarchy is not the hierarchy of n -categories (sets, categories, 2-categories, ...) but that of n -orders (1, truth values, orders, 2-orders (Ord-enriched categories), 3-orders, ...).

The first point has been defended, among others, by constructivists (for example [Roger Apéry](#)) and by [Michael Makkai](#). It is also usually the case in the formalisations of category theory in proof assistants that the definition of category does not include an equality relation on objects.

The n -order hierarchy is the natural extension (by iterated enrichment) of the [\(-2\)- and \(-1\)-categories](#) (i.e. 1 and the truth values) discovered by Toby Bartels and James Dolan and also follows from the first point: for example, the right notion of “1-category of dimension 0” cannot be a category whose arrows are all identities (i.e. sets, in the traditional way), since one can compare arrows only between two fixed objects, but rather a category where all arrows between fixed objects are equal (i.e. orders). Since that time, this hierarchy appeared in the [Lectures on n-Categories and Cohomology](#) by Baez and Shulman and on the [nLab](#), but surprisingly it seems not to have been adopted by working categorists (and mathematicians).

The second point has given its (current) name to the blog: *Higher-dimensional order theory*, which I consider to be a better name for what is usually called *category theory*.